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PROBLEMS.

9. Proposed by CHARLES E. MYERS, Canton, Ohio.

A ladder inclined at an angle of 60° to the horizon rests with one end on a *rough* pavement and the other against a *smooth* vertical wall. If the coefficient of friction between the foot of the ladder and the pavement is $\frac{1}{6}\sqrt{3}$, to what height can a man ascend before the ladder will begin to slip?

10. Proposed by G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

A paraboloid floats in a liquid which fills a fixed paraboloidal shell; both the paraboloid and the shell have their axes vertical and their vertices downward: the latus rectum of the paraboloid and shell are equal, and the axis of the shell is m times that of the paraboloid. If the paraboloid be pressed down until its vertex reaches the vertex of the shell, so that some of the liquid overflows, and then released, it is found that the paraboloid rises until it is just wholly out of the liquid, and then begins to fall. Prove that (1) the densities of the paraboloid and liquid are in the ratio $2[m^2+m+1-(m+1)\sqrt{(m^2-1)}]\ :\ 3\sqrt{(m+1)\div(m-1)}$, the free surface of the liquid being supposed to remain horizontal throughout the motion; and (2) if cone and conical be used, the ratio is $3[m^4-1-(m^3-1)\sqrt[3]{(m^3-1)}]:4\sqrt[3]{(m^3-1)}$, the vertical angles being equal.

Solutions to these problems should be received on or before May 1st.

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DIOPHANTINE ANALYSIS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTION TO THE CELEBRATED INDETERMINATE EQUATION.

$$x^2-Ny^2=\pm1.$$

By A. H. BELL Hillsboro, Illinois.

[Continued from February Number.]

It will already be noted, that whenever the denominator of y in (A) or the converging difference (as it is called) becomes $+1$, then the denominator $m^2N-n^2=+1$, by changing signs etc. is $n^2-Nm^2=-1$, and when the converging difference is -1, gives $n^2-Nm^2=+1$. The n and m in both cases occupy the position of x and y, in the original equation. Formula (A) giving the second higher values as x_2 and y_2 these answering always the $+1$ condition, and we now have the germ, of the formula as given by Legendre, to obtain the 2nd values after the 1st are obtained. This seems to be the extent to which the higher values of x and y were carried, except by actual computations.

$x^2-Ny^2=-1$, this class of numbers N, can always be known by this converging difference, being +1. They are also known when the number of fractions in the cycle, are odd, from $\sqrt{N}$.

Again the majority of them can be recognized, when of the form of a^2+1, or $4m+1$, without regard to prime numbers especially.

Formula (A), in the value given for y, the denominator m^2N-n^2 must be a *factor* of $2\times m\times N$ and the factor required from N must also be one of n. When m is made successively 1, 2, 3, etc. and substituted in m^2N-n^2, the resulting nearest values of n are called the + or − *Convergent Difference*. The small differences appear in cycles, and they can be developed in a series of two differences, while the corresponding m and n values proceed in an arithmetical series. Then we have a triple series; and a term is m, n, and convergent difference. In making $m=$to 1, 2, 3, etc. and a factor of $2\times m\times N$ etc. not being found, the first comparatively small + or − convergent difference of the advance terms is selected and multiplied by 2 becomes D_2, of the second order of differences, for the convergent diff. series. The m and n belonging to the selected convergent difference constitute the *common difference* for the two series of m's and n's, up to this time for every value given to m and the resulting two n's with their two differences give two terms with the same m.

Two terms of the Triple Series are now selected from these sets of preceding terms, so that when the m's and n's are added together will equal m and n already chosen for the common differences. In commencing a series, the two terms will always have contrary signs, the term having the same sign of the D_2 selected will constitute the 1st term of the Triple Series. With what is now given the convergent difference series is developed until it changes signs, and a limit is reached. The number of terms now being found, the m and n series is then carried forward, and if at the limit reached the convergent differences fail to become a factor required; then a New Series is made using the last term for the D_2 etc. provided the two last differences are nearly equal. If not, use the term having the small difference and 2× convergent difference$=D_2$. Proceeding the same as at first and continuing the operation, until the required factor is found, when the values of x and y are quickly found in whole numbers.

No 1. Example. Given $x^2-13y^2=\pm1$, to find x and y in integers.

Value N	m^2	n^2 + Diff.	n^2 − Diff.	No. Terms 1	2	3	4	
					+2			
13	$=13\times1^2$	$=3^2+4$	$=4^2-3$	$m=1$	+1	3	5	and $y=180$
					+7			
52	$=13\times2^2$	$=7^2+3$		$n=3$	+4	11	18	$x=649$
Convergent series of Diff.				+4	−3	−4	+1	

1st order of differences = ... $D_1=-7-1+5$

The conv. diff. for, $m=2$, and $n=7$, $=+3$ and $\times 2=D_2=+6+6+6$

Proof: $421201-13\times180^2=+1$

The conv. diff. being +1, x and y are 18 and 5, as shown for the −1 condition.

No. 2. Example: Given $x^2-19y^2=1$, to find x and y.

Value N	m^2	n^2+Diff.	n^2−Diff.
$19=19\times$	1^2	$=4^2+3$	$=5^2-6$
$76=19\times$	2^2	$=8^2+12$	$=9^2-5$
$171=19\times$	3^2	$=13^2+2$	

$$\text{This gives at once}\left.\begin{array}{r}m=3\\ n=13\\ \text{Diff.}\quad +2\end{array}\right\}\text{and in}(A)\text{ we have }\begin{array}{r}y=39.\\ x=180.\end{array}$$

Proof: $28900-19\times 1521=1$.

No. 3. Example: $x^2-31y^2=1$, to find x and y.

Value N m^2 n^2+ Diff. n^2−Diff. Terms No. 1 2 3

$$\left.\begin{array}{l}31=31\times 1=5^2+\ 6^2=6-5\\ \\ 124=31\times 2^2=11^2+\ 3\\ 279=31\times 3^2=\ldots\ldots\ldots\ldots 17^2-10\\ 496=31\times 4^2=22^2+12\\ \\ 775=31\times 5^2=\ldots\ldots\ldots\ldots 28^2-9\end{array}\right\}\quad\begin{array}{lrrr} & & & +5\\ \text{Take } m & =3 & +\ 2 & 7\\ & & & +28\\ n & =17 & +\ 11 & 39\\ \text{Diff.} & -10 & +3 & -2\\ D_1 & & +13 & -5\\ D_2 & & -18\ldots & -18\end{array}\left.\begin{array}{l}\\ \\ \\ \end{array}\right\}\text{give }\begin{array}{r}y=273\\ x=1520\end{array}$$

(TO BE CONTINUED.)

1. **Proposed by EARL D. WEST, West Middleburg, Logan County, Ohio.**

It is required to divide a given square number into two such parts that each part will be a square number.

I. **Solution by Hon. JOSIAH H. DRUMMOND, LL. D., Portland, Maine.**

No general solution can be given because there are *comparatively* few square numbers which are severally equal to the sum of two other squares. The well known equation $(2pq)^2+(p^2-q^2)^2=(p^2+q^2)^2$, in which $2pq$, p^2-q^2, and p^2+q^2 represent the sides of right triangles, is believed to include the cases in which the sum of two squares equals a third square. If this is true, it follows that the square root of the given square $(p^2+q^2)^2$ must also be the sum of two squares, p^2+q^2; and we have p^2+q^2=the square root of the given number to find $(2pq)^2$ and p^2-q^2, p being $>q$. If this can be done only by trial let us determine the limits within which the trials may be confined. Let m be the square root of the given number, and p and q may be any part positive numbers the sum of whose squares$=m$. Also, if any factor of m is equal to the sum of two squares, say s^2 and t^2, p and q may be s and t multiplied, respectively, by the corresponding factor, but, if any value of p is not prime to the corresponding value of q, the value obtained by some of the processes may be the same as the value obtained by another process. If $p^2+q^2=m$, remembering that $p>q$, we have $p<\sqrt{m}$ and $p<\sqrt{\frac{m}{2}}$. Take $m=82$, then $p<10$ and $p>6$; that is, p may be 7, 8, or 9, but $=7$ or 8 gives impossible values for q; while $p=9$, gives $q=1$. Then $2pq=18$ and $(p^2-q^2)=80$. Hence 18^2 and 80^2 are the numbers, and we have $18^2+80^2=82^2$. But $82=2\times 41$; then $s<7$ and $s>4$, and so may be 5